Decentralized H2 Optimization of Rotational Inertia Double-tuned Mass Dampers

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ABSTRACT: The rotational inertia double-tuned mass damper has been proposed and developed with the goal of improving upon the passive structural control performance of traditional tuned mass dampers. However, the effectiveness of these devices is highly dependent on the optimized stiffness and damping parameters of the device, as well as the amount of rotational inertia mass utilized. This optimization problem can be very complicated, particularly when the primary system is damped. Decentralized H2 optimization, which considers the effects of the supplemental passive damper on the structure as a control force, has been developed previously for the tuned mass damper. This paper presents the development of a decentralized H2 formulation for the rotational inertia double-tuned mass damper and then proposes a numerical optimization procedure designed to find optimum design values for this device.

1. INTRODUCTION

Tuned mass dampers (TMDs) have been widely studied as reliable passive control devices (Frahm 1909; Warburton 1982). With the goal of improving the effectiveness of TMDs, rotational inertia double-tuned mass dampers (RIDTMDs) have been proposed and investigated recently (Garrido et al. 2013; Hu and Chen 2015; Javidialesaadi and Wierschem 2017a; b, 2018). In a RIDTMD, the dashpot of a TMD is replaced by a tuning spring connected in series to a dashpot and an inerter, which are connected in parallel. The inerter is a recently proposed mechanical device, which provides a large effective inertia mass utilizing a small physical mass (Smith 2002). The RIDTMD shows superior performance in the reduction of the maximum dynamic magnification factor and RMS in comparison to TMDs (Garrido et al. 2013; Hu and Chen 2015). In previous investigations of the RIDTMD, numerical and exact optimization methods have been used to determine optimum design values (Garrido et al. 2013; Hu and Chen 2015). However, in these previous studies, the primary structure is considered undamped, which is not realistic. This paper proposes an optimum design procedure for the RIDTMD attached to a damped single-degree-of-freedom (SDOF) primary structure.

2. PROPOSED METHOD

Figure 2 shows the RIDTMD attached to a damped SDOF primary structure subjected to random force excitation. Considering the displacement of the primary structure as the system output, the equation of motion of this system can be found in (Garrido et al. 2013). The H2 norm of the system can be evaluated by solving the Lyapunov equation of the following governing equation:

$$\dot{x} = A_c x + B_c w; z = C_c x + D_c w \tag{1}$$

In order to optimize the RIDTMD with the decentralized control methodology, the springs and damper reactions can be considered as control forces. Figure 2 shows the diagram of the decentralized control method. In this figure, the feedback gain is decentralized as a matrix with parameters to be optimized.

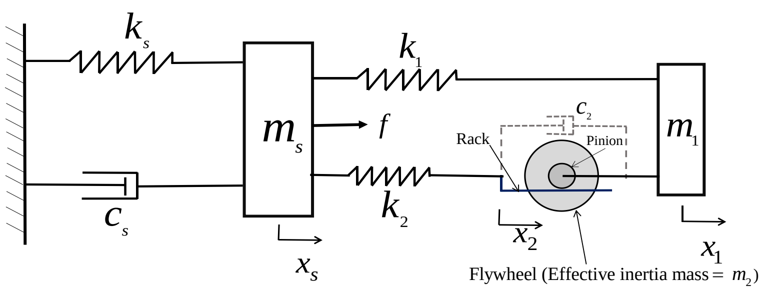

Figure 1: Rotational inertia double-tuned mass damper

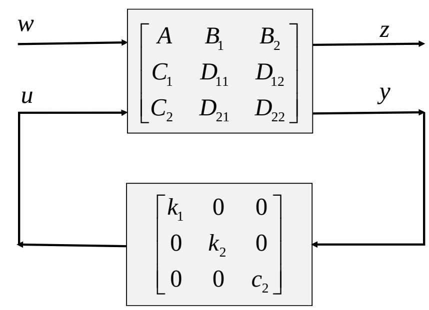

Figure 2: Block diagram of the decentralized control method

Considering the control forces as:

$$u_1 = k_1\left(x_1 - x_s\right); u_2 = k_2\left(x_2 - x_s\right); u_3 = c_2\left(\dot{x}_1 - \dot{x}_2\right) \tag{2}$$

the governing equation can be written as follows:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_1 + m_2 & -m_2 \\ 0 & -m_2 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_s \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_s & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}_s \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_s & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_s \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} f + \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{3}$$

Eq. (3) can be written in the form $\mathbf{M_q \ddot{q} + C_p \dot{q} + K_p q = B_f f + B_u u}$, where $\mathbf{q} = \left[\mathbf{x_s} \quad \mathbf{x_1} \quad \mathbf{x_2}\right]'$. Defining the state vector of the system as $x = \left[\mathbf{q} \quad \dot{\mathbf{q}}\right]'$, the governing equation can be written $\mathbf{\dot{x} = Ax + B_1 w + B_2 u}$, where $\mathbf{w} = \left[f, 0\right]'$ and

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{-M_q^{-1}K_q} & \mathbf{-M_q^{-1}C_q} \end{bmatrix}; \mathbf{B_1} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{-M_q^{-1}B_d} & \mathbf{0} \end{bmatrix}; \mathbf{B_2} = \begin{bmatrix} \mathbf{0} \\ \mathbf{-M_q^{-1}B_f} \end{bmatrix} \tag{4}$$

The system output, the displacement of the primary structure as the output, can be written as

$$\mathbf{z = C_1 x + D_{11} w + D_{12} u}; \quad \text{where} \quad \mathbf{C_1} = \left[1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\right]; \mathbf{D_{11}} = \left[1,0,0\right]; \mathbf{D_{12}} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

The control force, given by Eq. (2), as a static feedback gain is

$$\mathbf{u} = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & c_2 \end{bmatrix} \mathbf{y} \tag{5}$$

where $\mathbf{y} = [x_1 - x_s, x_2 - x_s, \dot{x}_1 - \dot{x}_s]' = \mathbf{C_2 x} + \mathbf{D_{21} w} + \mathbf{D_{22} u}$; $\mathbf{D_{12}} = \mathbf{0}; \mathbf{D_{22}} = \mathbf{0}$ and $\mathbf{C_2}$ are obtained from the state equations.

With the decentralized feedback $u = Fy$, the closed loop system is given by

$$\begin{bmatrix} \mathbf{A_c} & \mathbf{B_c} \\ \mathbf{C_c} & \mathbf{D_c} \end{bmatrix} = \begin{bmatrix} \mathbf{A} + \mathbf{B_2 F C} & \mathbf{B_1} + \mathbf{B_2 F D_{21}} \\ \mathbf{C_1} + \mathbf{D_{12} F C_2} & \mathbf{D_{11}} + \mathbf{D_{12} F D_{21}} \end{bmatrix} \tag{6}$$

Considering $\mathbf{D_{11}} + \mathbf{D_{12} F D_{12}} = \mathbf{0}$ (finite H2 norm) and utilizing the Lyapunov equation, the H2 optimization problem can be written as follows:

$$\min_F \|H_2\|^2 = \text{trace}\left(\left(\mathbf{B_1} + \mathbf{B_2 F D_{21}} \right)' \left(\mathbf{B_1} + \mathbf{B_2 F D_{21}} \right) \right) \tag{7}$$

Subject to the following Lyapunov equation

$$\mathbf{K}\left(\mathbf{A} + \mathbf{B_2 F C_2} \right) + \left(\mathbf{A} + \mathbf{B_2 F C_2} \right)' \mathbf{K} + \left(\mathbf{C_1} + \mathbf{D_{12} F C_2} \right)' \left(\mathbf{C_1} + \mathbf{D_{12} F C_2} \right) = \mathbf{0} \tag{8}$$

where K is the observability Grammian. The optimization problem of Eq. (7), subject to Eq. (8), can be solved by utilizing gradient-based optimization methods. In this paper, the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm, which is an unconstrained optimization algorithm, is adapted for this solution. In order to ensure positive resulting design values, $\mathbf{F} \odot \mathbf{F}$ can be used in the optimization instead of $\mathbf{F}$. It is observed that utilizing the results of the undamped system as the initial points (Javidialesaadi and Wierschem 2018) leads to rapid convergence of the RIDTMD parameters for this damped system.

3. RESULTS

Considering secondary mass ratios equal to 1% and 5% and damping of the structure equal to 2.5% and 5%, the responses of the optimized TMD and RIDTMD are shown in Figure 3.

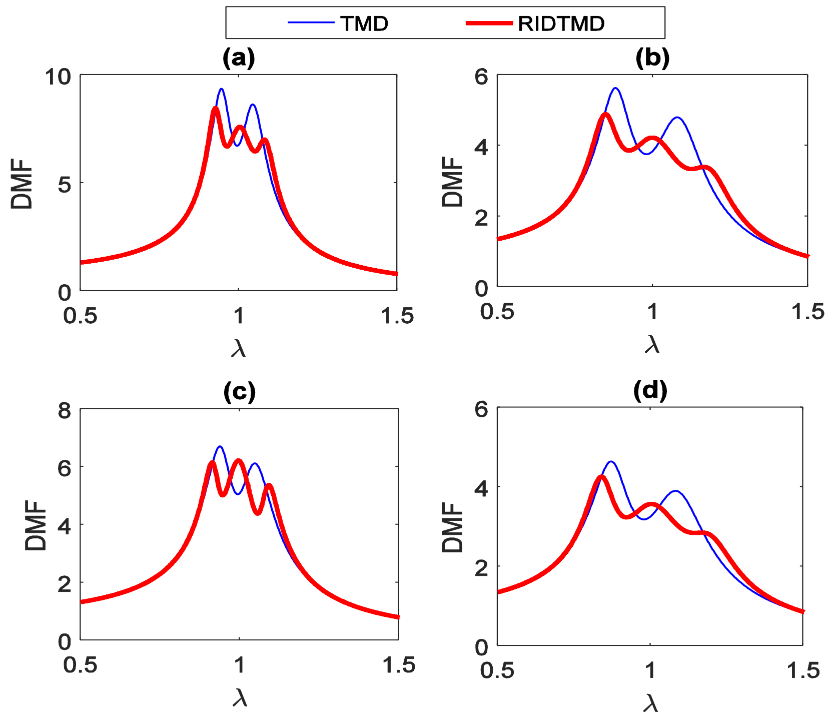

Figure 3: TMD and RIDTMD optimum response (a) m2=1%, damping=2.5%, (b) m2=5%, damping=2.5%, (c) m2=1%, damping=5%, and (d) m2=5%, damping=5%,

It is observed from these results that the optimum RIDTMD provides a 7% to 10% reduction in dynamic magnification factor compared to the TMD. In addition, the frequency response of RIDTMD shows three peaks, which indicates the effect of the third degree-of-freedom.

4. CONCLUSIONS

The optimum design of the RIDTMD has been considered previously; however, only with undamped primary systems. In this paper, an H2 decentralized method for optimum design of the RIDTMD when the primary structure is damped has been considered. This H2 decentralized method is developed and formulated for the RIDTMD and solved utilizing a gradient-based decentralized control method.

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